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THE USE OF HELICOPTERS IN UNDERWAY REPLENISHMENT.

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THESIS

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ABSTRACT

This is a model of the underway replenishment of a task group by a single supply ship which is capable of transferring logistic items by helicopter as well as by the connected method. The model considers two cases where replenishment time is minimized. In one case all ships break away from the supply ship when refueling is complete. In the other case, the CVA remains alongside until all her requirements have been satisfied while the remaining ships break away when refueling is complete.

The replenishment operation discussed deals specifically with a task group composed of one CVA, three DLG's and three DD's being rearmed and refueled by a single AOE. The specific portions of ordnance received via connected replenishment and vertical replenishment for each ship are the unknown quantities to be determined, while the transfer rates, refueling times, and total ordnance requirement are assumed to be known. A modified linear programming technique is used to determine an optimal employment of helicopters so that vertical replenishment time, and so the total replenishment time, is minimized. Operational data is used to establish the transfer rates and the individual ship requirements.

TABLE OF CONTENTS

4

LIST OF TABLES

LIST OF ILLUSTRATIONS

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I. INTRODUCTION

The mobility and flexibility of the Navy has been made possible by the ability to sustain itself at sea for extended periods of time, through the underway replenishment (UNREP) of needed logistic items such as food, fuel, and ordnance. Combatant ships are equipped to carry food and stores sufficient for from 45 to 90 days. Fuel and ordnance requirements are largely dependent upon the type of operations in which they are engaged. Under combat conditions, combatants may be expected to require replenishment of fuel and ordnance every three days.

Underway replenishment can be accomplished in essentially two different ways. The most frequently used method is the connected replenishment (CONREP) which involves the transfer of logistic items via rigs connecting the supply ship and the customer ship. The other method is vertical replenishment (VERTREP) which involves the use of helicopters. Fuel is the only item which is not adaptable to VERTREP. Often a combination of CONREP and VERTREP is applied to carry out the replenishment operations.

Underway replenishment must be accomplished with minimum increase in vulnerability and diversion from performance of the primary mission of the combatants. Therefore, a primary objective is the safe delivery of the required logistic items in a minimum of time. In an attempt to reduce replenishment time, the Navy has made studies in the areas of new designs in supply ships, improved delivery techniques, and more extensive training of personnel.

The planning phase of underway replenishment represents an important factor in the overall efficiency of the operation. It is imperative that the decision makers have a thorough knowledge of the capabilities and limitations of the ships involved. Although recent studies of the replenishment operation have been conducted by McCullough [1], Gordon and Copes [2], Waggoner [3], Besecker [4], and Patterson [5], the role of vertical replenishment has not been addressed in any of these studies.

II. THE REPLENISHMENT OPERATION

Underway replenishment is an operation in which logistic items are transferred at sea from supply ships to customer ships. UNREP can be accomplished in different ways, depending upon the customer requirements and the supply ship capabilities. Connected replenishment (CONREP) involves the intership horizontal transfer of bulk fluids and solids via rigs connecting the supply ship and the customer. The CONREP procedure calls for the customer ship to assume a position about 500 yards astern of the supply ship and make an approach on the supply ship until she is alongside at the required distance of 60 to 200 feet, depending upon the type of ship. The supply ship then sends over her rigs. After the rigs have been connected, the transfer of material takes place.

Vertical replenishment (VERTREP) involves the use of helicopters. The helicopter hovers over the supply ship, lifts the load, and transfers it to the deck of the customer ship. After depositing the load aboard the customer ship, the helicopter returns to the supply ship for another load. The solid cargo is transferred either in containers, in conventional pallets, or in standard cargo nets. Missiles are transferred in wheeled dollies.

Some of the more recently built Navy support ships are designed to transfer several logistic items simultaneously. One of these is the AOE, which is the only type of supply ship discussed in this paper. It is designed to carry fuel and ordnance, plus limited quantities of provisions and stores. The AOE is capable of conducting a CONREP with two ships simultaneously, one along the port side and the other along the starboard side. Since it is equipped with two replenishment

helicopters, the AOE may VERTREP up to two additional ships, simultaneously with the CONREP.

With the advent of vertical replenishment came problems of how to integrate this concept into replenishment operations. The heavy ordnance requirements which have resulted from the South East Asia conflict have quickly shown that vertical replenishment is an operational concept with a bright future. The type of operations being conducted in that area, wherein the AOE is usually replenishing only a CVA and two escorts at any one time, has led to a relatively loose policy of VERTREPing ships. The method of replenishment has previously been determined by the CO of the AOE asking each ship whether she would like to VERTREP or CONREP. The ships must come alongside anyway to refuel. The future concept of task group operations envisions one CVA and six DLG/DX's replenishing every three days from an AOE. With a task group of this size, arbitrary deployment of helicopters is not feasible.

The author is aware of a computer simulation model prepared by PMS-390 of the Naval Ship Systems Command involving the replenishment of a carrier task group which is composed of a CVA (attack carrier) and its accompanying escorts. The replenishing ships include either an AOE or an AO and AE combination. The replenishment is simulated for a case with minimum alongside time and a case with minimum replenishment time. To minimize alongside time the carrier breaks away from the AOE or AO when fueling has been completed, if helicopters are available. To minimize replenishment time, the carrier remains alongside the AOE or AE until replenishment has been completed. In either case, the escort vessels break away when refueling has been completed, if there are helicopters available to the escorts. These two cases are discussed in the following chapters.

This paper represents a first attempt to analytically model a replenishment operation where both CONREP and VERTREP procedures are used. The objective is to consider different ways of employing helicopters and the subsequent effect this has on the replenishment time. This model uses relatively simple methods to help better understand the options and choices available to the officer in command of the replenishment operation. It is hoped that this work may lead to further development that may eventually assist the forces afloat to plan UNREPs more efficiently.

III. FORMULATION OF THE PROBLEM

Various combinations of supply ships and customers occur in replenishment operations. This paper deals specifically with the case of one supply ship and seven customers. The problem is to minimize the time required for this single supply ship to replenish the seven customers. Two cases are discussed:

(1) Minimize the total replenishment time of the task group, keeping all seven customers alongside only until refueling is completed.

(2) Minimize the total replenishment time of the task group, with the CVA remaining alongside until all of her replenishment requirements have been satisfied. The escorts remain alongside only until refueling is completed.

Variations to the cases arise according to the number of helicopters available and how these helicopters are used. Each customer can be served either by one, two, or three servers. The supply ship is capable of simultaneously rearming and refueling two ships via CONREP plus rearming one or two additional ships via VERTREP, depending upon the number of helicopters available. The supply ship is equipped with two UH - 46 replenishment configured helicopters which are designed to carry ordnance, provisions, stores, and personnel.

When the replenishment begins, the seven customers are divided into two groups, with three customers waiting to go along the portside of the supply ship, and four customers waiting to go along the starboard side of the supply ship. In all cases considered, the CVA is the first customer to be serviced and she always goes along the port side of the supply ship. The remaining order alongside is also predetermined.

In Case I, if a customer has completed refueling but the ordnance requirement has not been satisfied, it breaks away; the remaining ordnance is VERTREPed. In Case II this same situation applies to all except customer 1; customer 1 remains alongside until all its requirements have been satisfied.

The UNREP is completed when all customers have fulfilled their requirements. Figure 1 represents the initial position of the ships and one possible deployment of the helicopters.

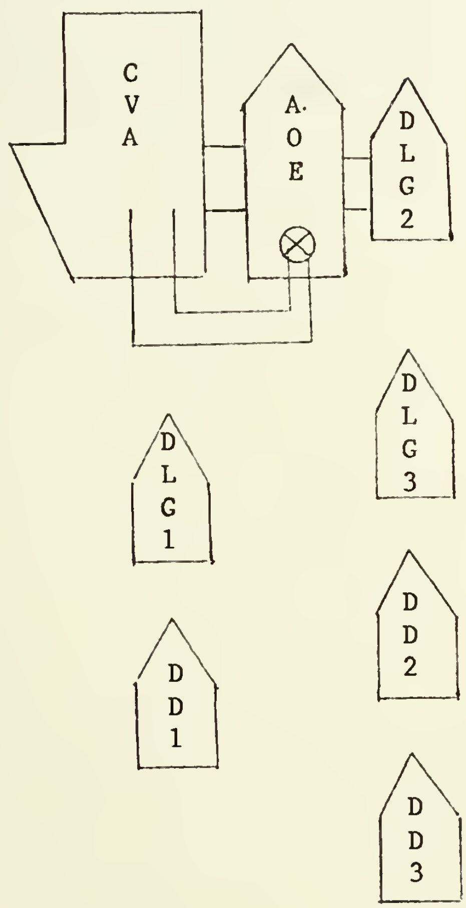

Figure 1. Initial Position for UNREP

The servers do not act independently. The amount of ordnance transferred by CONREP to an individual customer is dependent upon the amount of fuel required. The amount of ordnance transferred by VERTREP is dependent upon how much ordnance was transferred by CONREP. A customer's "service time" for CONREP is defined to be the transfer time, plus the rig-unrig time, plus the approach time. The service time for VERTREP is defined to be just the transfer time.

Estimates of fuel and ordnance requirements are sent to the AOE prior to the replenishment. This paper assumes that the estimates of fuel requirements are accurate and therefore the individual fuel transfer times are known before the UNREP begins.

IV. THE MODEL

A. GENERAL DISCUSSION

The transfer of logistic items from a single supply ship to M customers is accomplished by CONREP and, if replenishment helicopters are available, by VERTREP. The customer ships are required to go alongside the supply ship to refuel. During the time the customers are alongside to refuel, they also replenish ordnance. If the time required to refuel is not sufficient for the ordnance requirement to be satisfied, then the customer must either remain alongside until all requirements are satisfied or break away and obtain the remaining ordnance by VERTREP.

The model represents an eight-ship replenishment operation, with one supply ship and seven customers. Seven customers are used because this is considered to be the normal size for a carrier task group. The customers include one CVA and six destroyer types (three DLG's and three DD's).

The model includes four servers, namely CONREP port side, CONREP starboard side, VERTREP helicopter 1, and VERTREP helicopter 2. More than four servers would require the introduction of another supply ship which would significantly alter this model.

The model investigates the minimization of total replenishment time by use of the supply ship's VERTREP capability. The first step of the model is to set up equations which express the total VERTREP time for each helicopter. Then the total VERTREP time for the whole replenishment operation is computed. This time is then compared to the port side and

starboard side CONREP times to arrive at the total replenishment time which is the maximum of VERTREP time, port side CONREP time, and starboard side CONREP time.

The following notation is used in the model:

X_{ij} is the unknown amount of ordnance to be transferred from server i to customer j for $i = 1,\ldots,4$; $j = 1,\ldots,7$.

X_{v1} is the unknown amount of ordnance to be transferred via VERTREP to customer 1, i.e., $X_{v1} = X_{31} + X_{41}$. This notation is used in the situation where both helicopters are VERTREPing simultaneously to customer 1.

T_{ij} is the amount of material transferred per unit time (The transfer rate) from server i to customer j, i.e., short tons per hour (st/hr) or barrels per hour (bbl/hr). The inverse transfer rate, $\frac{1}{T_{ij}}$, is the time required to transfer a unit amount from server i to customer j. This quantity is used when computing CONREP and VERTREP times.

T_{v1} is the combined transfer rate of two helicopters when they are VERTREPing simultaneously to customer 1. The data available from the fleet operations for a two helicopter simultaneous VERTREP indicates that the combined rate is less than the sum of the individual rates. Therefore, the model assumes that $T_{v1} \leq T_{31} + T_{41}$.

A_j is the amount of ordnance required by customer j.

F_j is the time required for customer j to refuel.

K_j is the sum of the approach time and the rig and unrig time for customer j. Approach time is defined as the time from which both customer and supply ship signal "ready to replenish" until the first line is secured. Rig time begins with the first line over between supply and customer ship;

it ends when transfer of material begins. Unrig time begins when transfer ceases: it ends when the customer departs from alongside the supply ship.

The labeling of servers and customers used in this chapter is listed below:

SERVER	#	CUSTOMER	#
CONREP PORT SIDE	1	CVA	1
CONREP STARBOARD SIDE	2	DLG 1	2
VERTREP HELICOPTER 1	3	DD 1	3
VERTREP HELICOPTER 2	4	DLG 2	4
		DLG 3	5
		DD 2	6
		DD 3	7

The order alongside remains the same for all examples. Customers 1, 2, 3 are served by server 1; customers 4, 5, 6, 7 are served by server 2. Servers 3 and 4 may serve any of the customers.

The values assigned to known quantities in the examples are based on operational data [6] for a three-day replenishment cycle.

B. CASES

Two cases are discussed in the model. In Case I, the problem is to minimize the total replenishment time of the carrier task group defined in Part A, keeping the seven customers alongside the supply ship only until the refueling operation is completed. The model development for Case I is broken down into four variations depending upon the number of helicopters available, the employment of the helicopters, and the helicopter transfer rates.

In Case II, the total replenishment time is again minimized, but this time customer 1 (CVA) remains alongside until all its replenishment requirements have been satisfied. During this time, customer 1 is also VERTREPed. The remaining six customers are alongside the supply ship only until the refueling operation is complete. Case II is actually different from Case I only when $T_{11}F_1 < A_1$, i.e., the amount of ordnance that can be transferred to customer 1 during the time allotted for its refueling is less than its total ordnance requirement. Case II is broken down into five variations, the first four being identical to those of Case I, and the fifth being a situation with no helicopters available.

The variations are presented using the following format for each variation:

a. Discussion of the Variation

b. The Solution Technique for Case I

c. Case I Example

d. The Solution Technique for Case II

e. Case II Example

1. Variation One

a. In this situation, two helicopters are available for VERTREP, and each helicopter may have a different transfer rate. In addition, customer 1 (the CVA) is VERTREPed simultaneously by both helicopters with a combined transfer rate of T_{v1}. The remaining six customers are restricted to the use of only one helicopter.

b. In Case I, the total VERTREP time for each helicopter is denoted by Z_1 and Z_2 respectively, where:

(1) $Z_1 = \frac{2}{T_{v1}} \frac{X_{v1}}{2} + \frac{1}{T_{32}} X_{32} + \frac{1}{T_{33}} X_{33} + \frac{1}{T_{34}} X_{34} + \frac{1}{T_{35}} X_{35} + \frac{1}{T_{36}} X_{36} + \frac{1}{T_{37}} X_{37}$

(2) $Z_2 = \frac{2}{T_{v1}} \frac{X_{v1}}{2} + \frac{1}{T_{42}} X_{42} + \frac{1}{T_{43}} X_{43} + \frac{1}{T_{44}} X_{44} + \frac{1}{T_{45}} X_{45} + \frac{1}{T_{46}} X_{46} + \frac{1}{T_{47}} X_{47}$

In view of the fact that customer 1 is being VERTREPed simultaneously by both helicopters, and each helicopter spends an equal amount of time with the customer, the amount of ordnance transferred by each helicopter must be the same, otherwise the VERTREP time will not be minimal. Since $X_{31} + X_{41} = X_{v1}$, this implies that $X_{31} = X_{41} = \frac{1}{2}X_{v1}$. The transfer rate for each helicopter when VERTREPing customer 1 is then effectively one-half the combined transfer rate, T_{v1}. Therefore, $\frac{2}{T_{v1}} \frac{X_{v1}}{2}$ represents the time required for each helicopter to VERTREP customer 1.

Similarly, $\frac{1}{T_{3j}} X_{3j}$ and $\frac{1}{T_{4j}} X_{4j}$ represent the times required for servers 3 and 4, respectively, to VERTREP customer j, with j ranging from 2 to 7.

The total VERTREP time, Z, for the replenishment if found by minimizing the maximum of Z_1 and Z_2, subject to the following constraints:

Each customer must receive its ordnance requirements:

(3) $X_{11} + X_{v1} = A_1$

(4) $X_{12} + X_{32} + X_{42} = A_2$

(5) $X_{13} + X_{33} + X_{43} = A_3$

(6) $X_{24} + X_{34} + X_{44} = A_4$

(7) $X_{25} + X_{35} + X_{45} = A_5$

(8) $X_{26} + X_{36} + X_{46} = A_6$

(9) $X_{27} + X_{37} + X_{47} = A_7$

Further, the alongside time for each customer must not exceed its refueling time:

(10) $X_{11} \leq T_{11} F_1$

(11) $X_{12} \leq T_{12} F_2$

(12) $X_{13} \leq T_{13} F_3$

(13) $X_{24} \leq T_{24} F_4$

(14) $X_{25} \leq T_{25} F_5$

(15) $X_{26} \leq T_{26} F_6$

(16) $X_{27} \leq T_{27} F_7$

Since only one of the two helicopters may VERTREP each of customers 2 through 7, the following conditions are imposed:

$$X_{3j} X_{4j} = 0 \qquad \text{for } j = 2,\cdots,7$$

Finally, the amount of ordnance transferred by the four servers to each customer is of course a non-negative quantity:

$$X_{1j} \geq 0 \quad \text{for } j = 1,2,3$$

$$X_{2j} \geq 0 \quad \text{for } j = 4,5,6,7$$

$$X_{3j}, X_{4j} \geq 0 \text{ for } j = 1,\cdots,7$$

Since the problem is to minimize total VERTREP time, the optimal solution would be to have all the X_{3j}'s and X_{4j}'s equal to zero and have $X_{12} = A_2$, $X_{13} = A_3$, $X_{24} = A_4$, $X_{25} = A_5$, $X_{26} = A_6$, $X_{27} = A_7$. However, since the constraints set an upper bound on the amount of ordnance transferred by CONREP, this is not always possible. If the total ordnance requirement for customer j, A_j, is less than or equal to the upper bound value in the appropriate inequality (10) - (16), then all the ordnance is transferred by CONREP. If the total ordnance requirement is greater than that upper bound value, then their resulting difference is transferred via VERTREP. Therefore the optimal amount of ordnance transferred by CONREP from server 1 to customer j is

$$X_{ij} = \min \left[(T_{ij}\ F_j)\ ,\ A_j \right] = C_j \text{ for } j = 1,2,3\ .$$

Similarly, the optimal amount of ordnance transferred by CONREP from server 2 to customer j is

$$X_{2j} = \min \left[(T_{2j}\ F_j),\ A_j \right] = C_j \text{ for } j = 4,5,6,7.$$

Since X_{11}, X_{12}, X_{13}, $X_{24,}$ $X_{25;}$ X_{26}, X_{27} have been computed, the constraints can be represented as follows:

$$X_{v1} = (A_1 - C_1)$$

$$X_{3j}\ X_{4j} = (A_j - C_j) \text{ for } j = 2,\ldots,7$$

$$X_{3j}\ X_{4j} = 0 \text{ for } j = 2,\ldots,7$$

$$X_{3j}\ , X_{4j} \geq 0 \text{ for } j = 1,\ldots,7.$$

One way the optimal solution to this problem may be obtained is to compute all possible combinations ($\frac{1}{T_{ij}} X_{ij}$ values for i = 3,4; j = 2,•••,7) in which the six customers can be VERTREPed by helicopter 1 or helicopter 2. Then find the maximum of Z_1' and Z_2' for each combination where:

$$Z_1' = \sum_{j \in J_1} \frac{1}{T_j} X_{3j}$$

$$Z_2' = \sum_{j \in J_2} \frac{1}{T_{4j}} X_{4j} ,$$

where J_1 is the set of customers VERTREPed by helicopter 1 for each combination, and J_2 is the set of customers VERTREPed by helicopter 2 for each combination. $J_1 \cap J_2 = \emptyset$ for each combination. Then find the minimum Z'* of all the maximums. Finally

$$\min Z = \frac{2}{T_{v1}} \frac{X_{v1}}{2} + Z'^{*} .$$

The number of combinations that must be computed if the transfer rates are not equal is 2^{n-1}, where n is the number of customers that are VERTREPed. If the transfer rates are equal ($T_{3j} = T_{4j}$) symmetry implies that only half i.e., 2^{n-2} of the combinations need to be computed. When is large, this procedure becomes unmanageable; even when $n = 6$ there are 32 combinations to compute for the unequal transfer rate situation, and 16 combinations to compute for the equal transfer rate situation. Consequently, a different solution technique is desirable.

The use of linear programming methods, with some modification, provides a convenient way to arrive at a optimal solution. Details of the linear programming technique are presented in Appendix A. The total replenishment time is found by taking the maximum of the following: total minimum VERTREP time Z*, total port side CONREP service time P, and total starboard side CONREP service time S, where

$$Z^* = \min \max (Z_1, Z_2)$$

$$P = \sum_{j=1}^{3} \left[\max \left(\frac{C_j}{T_{1j}}, F_j\right) + K_j \right]$$

$$S = \sum_{j=4}^{7} \left[\max \left(\frac{C_j}{T_{2j}}, F_j\right) + K_j \right] .$$

Total replenishment time (TRT) = max (Z*,P,S).

The equations for P and S contain only known quantities; therefore when minimizing Z, if its value becomes less than either P or S, the minimum total replenishment time has been found. However, under these conditions it may be possible to further reduce VERTREP time without affecting TRT. This reduction is desirable, since it makes for a more economical use of the helicopters.

c. Example 1: Find the minimum total replenishment time of a seven-ship task group if all ships remain alongside the supply ship only until refueling has been completed. Two helicopters are used for VERTREP. The CVA is VERTREPed by both helicopters simultaneously. The remaining six ships are restricted to the use of one helicopter. Known quantities are:

CONREP transfer rates

$T_{11} = 150$ st/hr

$T_{1j} = 25$ st/hr for $j = 2,3$

$T_{2j} = 25$ st/hr for $j = 4,5,6,7$

VERTREP transfer rates

$T_{3j} = 18$ st/hr for $j = 1,\cdots,7$

$T_{4j} = 24$ st/hr for $j = 1,\cdots,7$

$T_{v1} = 30$ st/hr

Approach-rig-unrig times

$K_1 = 0.5$ hrs.

$K_j = 0.4$ hrs. for $j = 2,\cdots,7$

Refueling times

$F_1 = 2.41$ hrs.

$F_2 = 1.20$ hrs.

$F_3 = 0.90$ hrs.

$F_4 = 1.20$ hrs.

$F_5 = 1.20$ hrs.

$F_6 = 0.90$ hrs.

$F_7 = 0.90$ hrs.

Individual ordnance requirements

$A_1 = 500$ st .

$A_2 = 60$ st .

$A_3 = 40$ st .

$A_4 = 70$ st .

$A_5 = 50$ st .

$A_6 = 30$ st .

$A_7 = 20$ st .

These values assigned to given quantities in the examples are based on operational data [6] for a three-day replenishment cycle.

Solution:

Set up the equations which express the ordnance requirements

$$X_{11} + X_{31} + X_{41} = 500 \text{ st.}$$

$$X_{12} + X_{32} + X_{42} = 60 \text{ st.}$$

$$X_{13} + X_{33} + X_{43} = 40 \text{ st.}$$

$$X_{24} + X_{34} + X_{44} = 70 \text{ st.}$$

$$X_{25} + X_{35} + X_{45} = 50 \text{ st.}$$

$$X_{26} + X_{36} + X_{46} = 30 \text{ st.}$$

$$X_{27} + X_{37} + X_{47} = 20 \text{ st.}$$

Establish the upper bound values on the CONREP of ordnance

$$X_{11} \leq (150)(2.41) = 362 \text{ st.}$$

$$X_{12} \leq (25)(1.20) = 30 \text{ st.}$$

$$X_{13} \leq (25)(\ .90) = 22 \text{ st.}$$

$$X_{24} \leq (25)(1.20) = 30 \text{ st.}$$

$$X_{25} \leq (25)(1.20) = 30 \text{ st.}$$

$$X_{26} \leq (25)(\ .90) = 22 \text{ st.}$$

$$X_{27} \leq (25)(\ .90) = 22 \text{ st.}$$

The total VERTREP times Z_1 and Z_2 for helicopters 1 and 2 respectively are given by

$$Z_1 = \frac{2}{30}\,\frac{X_{v1}}{2} + \frac{1}{18}\,X_{32} + \frac{1}{18}\,X_{33} + \frac{1}{18}\,X_{34} + \frac{1}{18}\,X_{35} + \frac{1}{18}\,X_{36} + \frac{1}{18}\,X_{37}$$

$$Z_2 = \frac{2}{30}\,\frac{X_{v1}}{2} + \frac{1}{24}\,X_{42} + \frac{1}{24}\,X_{43} + \frac{1}{24}\,X_{44} + \frac{1}{24}\,X_{45} + \frac{1}{24}\,X_{46} + \frac{1}{24}\,X_{47}$$

The resulting programming problem may be stated as follows:

$$\min Z = \max (Z_1, Z_2)$$

subject to

$$X_{v1} = (500 - 362) = 138 \text{ implies } X_{31} = X_{41} = 69 \text{ st.}$$

$$X_{32} + X_{42} = (\ 60 - 30) = 30 \text{ st.}$$

$$X_{33} + X_{43} = (\ 40 - 22) = 18 \text{ st.}$$

$$X_{34} + X_{44} = (\ 70 - 30) = 40 \text{ st.}$$

$$X_{35} + X_{45} = (\ 50 - 30) = 20 \text{ st.}$$

$$X_{36} + X_{46} = (\ 30 - 22) = 8 \text{ st.}$$

$$X_{37} + X_{47} = (\ 20 - 20) = 0$$

$$X_{3j}\, X_{4j} = 0 \quad \text{for } j = 2,\cdots,7$$

$$X_{3j}, X_{4j} \geq 0 \quad \text{for } j = 1,\cdots,7.$$

Using the solution technique outlined in Appendix A, the optimal solution is

$$\begin{array}{ll} X_{31} = 69 & X_{41} = 69 \\ X_{32} = 30 & X_{42} = 0 \\ X_{33} = 0 & X_{43} = 18 \\ X_{34} = 0 & X_{44} = 40 \\ X_{35} = 20 & X_{45} = 0 \\ X_{36} = 0 & X_{46} = 8 \\ X_{37} = 0 & X_{47} = 0\ . \end{array}$$

Total optimal VERTREP times for helicopters 1 and 2 are

$$Z_1{*} = \frac{1}{15}\ (69) + \frac{1}{18}\ (30) + \frac{1}{18}\ (20) = 7.38 \text{ hrs.}$$

$$Z_2{*} = \frac{1}{15}\ (69) + \frac{1}{24}\ (18) + \frac{1}{24}\ (40) + \frac{1}{24}\ (8) = 7.35 \text{ hrs.}$$

Therefore, $Z* = Z_1* = 7.38$ hrs.

Total port side and starboard side CONREP service times are

$$P = 2.41 + 1.20 + 0.90 + 1.30 = 5.81 \text{ hrs.}$$

$$S = 1.20 + 1.20 + 0.90 + 1.60 = 5.80 \text{ hrs.}$$

Therefore, $TRT = \max(Z*, P, S) = Z* = 7.38$ hrs.

d. In Case II, one change is the removal of the constraint which requires customer 1 to remain alongside only until its refueling has been completed (equation (10)). The amount of ordnance transferred by CONREP to customer 1 may increase because customer 1 remains alongside until all its requirements have been satisfied. In order to insure that this time alongside is minimized, customer 1 is VERTREPed during this time. Therefore, $\frac{X_{v1}}{T_{v1}} = \frac{X_{11}}{T_{11}} + K_1$ where $\frac{X_{v1}}{T_{v1}}$ is the VERTREP transfer

time for customer 1 and $\frac{X_{11}}{T_{11}} + K_1$ is the CONREP service time for customer 1. This CONREP service time is the sum of the transfer time plus the approach-rig-unrig time. Solving for X_{v1} and substituting into equation (3) gives the following result: $X_{11} + T_{v1}(\frac{X_{11}}{T_{11}} + K_1) = A_1$. Solving for X_{11}, the total amount of ordnance to be CONREPed is found to be

$$X_{11} = \frac{A_1 - T_{v1}K_1}{(1 + \frac{T_{v1}}{T_{11}})} .$$

The total amount of ordnance to be VERTREPed to customer 1 is

$$X_{v1} = T_{v1}(\frac{X_{11}}{T_{11}} + K_1) .$$

The total VERTREP times for helicopters 1 and 2 are

$$Z_1 = \frac{2}{T_{v1}} \frac{X_{v1}}{2} + \frac{1}{T_{32}} X_{32} + \frac{1}{T_{33}} X_{33} + \frac{1}{T_{34}} X_{34} + \frac{1}{T_{35}} X_{35} + \frac{1}{T_{36}} X_{36} + \frac{1}{T_{37}} X_{37}$$

$$Z_2 = \frac{2}{T_{v1}} \frac{X_{v1}}{2} + \frac{1}{T_{42}} X_{42} + \frac{1}{T_{43}} X_{43} + \frac{1}{T_{44}} X_{44} + \frac{1}{T_{45}} X_{45} + \frac{1}{T_{46}} X_{46} + \frac{1}{T_{47}} X_{47} .$$

The only change from the Case I solution is that the amount VERTREPed to customer 1 is less. The employment of helicopters to the remaining six customers is the same as that used in Case I. Therefore, min Z = $\frac{2}{T_{v1}} \frac{X_{v1}}{2} + Z'^*$ where Z'^* is the value obtained in Case I.

TRT = max(Z^*,P,S) where

$$P = \frac{X_{11}}{T_{11}} + \sum_{j=2}^{3} \max\left(\frac{C_j}{T_{1j}}, F_j\right) + \sum_{j=1}^{3} K_j$$

$$S = \sum_{j=4}^{7} \left[\max\left(\frac{C_j}{T_{2j}}, F_j\right) + K_j\right] .$$

e. Example 2: Find the minimum total replenishment time of a seven ship task group, if the CVA remains alongside the supply ship until her requirements have been satisfied. The remaining ships break away when refueling has been completed. Two helicopters are used for VERTREP. The CVA is VERTREPed by both helicopters simultaneously. After the CVA has been VERTREPed, the remaining ships are VERTREPed by one helicopter. Known quantities are:

CONREP transfer rates VERTREP transfer rates Approach-rig-unrig time Refueling time Ordnance requirements	Same as in example 1

Solution:

The CVA ordnance requirement is expressed by $X_{11} + 30\left(\frac{X_{11}}{150} + 0.5\right) = 500$. Solving for the amount to be transferred by CONREP, $X_{11} = 404$ st. Therefore the amount to be transferred by VERTREP is $X_{v1} = 30\left(\frac{404}{150} + 0.5\right) = 96$ st. The remaining equations which express ordnance requirements and upper bound values for CONREP are identical to those given in Example 1.

Keeping the CVA alongside longer decreases the total VERTREP time but does not change the helicopter employment. Z_1 and Z_2 are both reduced by an equal amount. The change in the amount of VERTREP time is found by taking the difference between the amount VERTREPed in Case I and the amount VERTREPed in Case II for customer 1 and multiplying by the inverse transfer rate $\Delta \frac{X_{v1}}{T_{v1}} = \frac{1}{30}(138 - 96) = 1.40$ hrs.

Total VERTREP times for helicopters 1 and 2 are

$$Z_1^* = \frac{1}{15}(48) + \frac{1}{18}(30) + \frac{1}{18}(20) = 5.98 \text{ hrs.}$$

$$Z_2^* = \frac{1}{15}(48) + \frac{1}{24}(18) + \frac{1}{24}(40) + \frac{1}{24}(8) = 5.95 \text{ hrs.}$$

Therefore, min $Z = Z_1^* = 5.98$ hrs.

CONREP service time increases on the portside because the CVA remains alongside until her requirements have been satisfied. The starboard side CONREP service time does not change.

$$P = \frac{404}{150} + 1.20 + 0.90 + 1.30 = 6.09 \text{ hrs.}$$

$$S = 5.80 \text{ hrs.}$$

$$TRT = \max(Z^*, P, S) = P = 6.09 \text{ hrs.}$$

2. Variation Two

a. In this situation, two helicopters are available and each helicopter may have a different transfer rate as in Variation One. This time, however, all customers are restricted to the use of only one helicopter.

b. In Case I, the total VERTREP time for each helicopter is given by:

$$Z_1 = \sum_{j=1}^{7} \frac{X_{3j}}{T_{3j}}$$

$$Z_2 = \sum_{j=1}^{7} \frac{X_{4j}}{T_{4j}}$$

The total optimal VERTREP time, Z^*, is found as in Variation One by minimizing the maximum of Z_1 and Z_2, subject to the following constraints:

$$X_{3j} + X_{4j} = (A_j - C_j) \quad \text{for } j = 1,\cdots,7$$

$$X_{3j}\, X_{4j} = 0 \quad \text{for } j = 1,\cdots,7$$

$$X_{3j}, X_{4j} \geq 0 \quad \text{for } j = 1,\cdots,7,$$

where C_j was defined in Variation One. The minimum Z value is found by applying the solution technique presented in Appendix A, or by computing all the possible combinations in which the seven customers can be VERTREPed by helicopter 1 or helicopter 2 as explained in Variation One. There exist 2^n possible combinations here if the transfer rates are not equal and 2^{n-1} combinations if the transfer rates are equal. TRT = max (Z^*,P,S) where

$$Z^* = \min \max (Z_1, Z_2)$$

$$P = \sum_{j=1}^{3} \left[\max \left(\frac{C_j}{T_{ij}}, F_j \right) + K_j \right]$$

$$S = \sum_{j=4}^{7} \left[\max \left(\frac{C_j}{T_{2j}}, F_j \right) + K_j \right] .$$

c. Example 3: Find the minimum total replenishment time of a seven ship task group, if all the ships remain alongside the supply ship only until refueling has been completed. Two helicopters are available but each ship is VERTREPed by only one helicopter.

Known quantities are:

CONREP transfer rates VERTREP transfer rates Approach-rig-unrig time Refueling time Ordnance requirements	Same as in Example 1

Solution:

The equations which express ordnance requirements and CONREP upper bound values are identical to those given in Example 1.

The total VERTREP times Z_1 and Z_2 for helicopters 1 and 2 respectively are given by

$$Z_1 = \frac{1}{18} X_{31} + \frac{1}{18} X_{32} + \frac{1}{18} X_{33} + \frac{1}{18} X_{34} + \frac{1}{18} X_{35} + \frac{1}{18} X_{36} + \frac{1}{18} X_{37}$$

$$Z_2 = \frac{1}{24} X_{41} + \frac{1}{24} X_{42} + \frac{1}{24} X_{43} + \frac{1}{24} X_{44} + \frac{1}{24} X_{45} + \frac{1}{24} X_{46} + \frac{1}{24} X_{47} .$$

The resulting programming problem is

$$\min Z = \max(Z_1, Z_2)$$

subject to

$$\begin{aligned}
X_{31} + X_{41} &= 138 \\
X_{32} + X_{42} &= 30 \\
X_{33} + X_{43} &= 18 \\
X_{34} + X_{44} &= 40 \\
X_{35} + X_{45} &= 20 \\
X_{36} + X_{46} &= 8 \\
X_{37} + X_{47} &= 0
\end{aligned}$$

$$X_{3j}\, X_{4j} = 0 \quad \text{for } j = 1,\cdots,7$$

$$X_{3j}, X_{4j} \geq 0 \quad \text{for } j = 1,\cdots,7 .$$

Using the solution technique outlined in Appendix A, the optimal solution is

$$\begin{aligned}
X_{31} &= 0 & X_{41} &= 138 \\
X_{32} &= 30 & X_{42} &= 0 \\
X_{33} &= 18 & X_{43} &= 0
\end{aligned}$$

$$X_{34} = 40 \qquad X_{44} = 0$$

$$X_{35} = 20 \qquad X_{45} = 0$$

$$X_{36} = 0 \qquad X_{46} = 8$$

$$X_{37} = 0 \qquad X_{47} = 0 \quad .$$

Total optimal VERTREP times for helicopter 1 and 2 are

$$Z_1^* = \frac{1}{18}(30) + \frac{1}{18}(18) + \frac{1}{18}(40) + \frac{1}{18}(20) = 6.00 \text{ hrs.}$$

$$Z_2^* = \frac{1}{24}(138) + \frac{1}{24}(8) = 6.08 \text{ hrs.}$$

So it follows that $Z^* = Z_2^* = 6.08$ hrs.

Total port side and starboard side CONREP service times are $P = 5.82$ hrs and $S = 5.80$ hrs.

Therefore, $TRT = \max(Z^*,P,S) = Z^* = 6.08$ hrs.

d. In Case II, the amount of ordnance transferred by CONREP to customer 1 may increase because customer 1 remains alongside until all its requirements have been satisfied. Customer 1 is VERTREPed during this time by the helicopter with the greater transfer rate. The greater rate is used to insure that customer 1 spends a minimum amount of time alongside. Using the smaller of the two rates would further increase the amount to be transferred by CONREP, and would therefore extend customer 1's alongside time.

The equation which expresses the ordnance requirement of customer 1 is

$$X_{11} + \max(T_{31},T_{41})\left(\frac{X_{11}}{T_{11}} + K_1\right) = A_1 \; .$$

Solving for X_{11}, the total amount of ordnance to be CONREPed is found to be

$$X_{11} = \frac{A_1 - \max(T_{31},T_{41})K_1}{\left(1 + \frac{\max(T_{31},T_{41})}{T_{11}}\right)} \quad .$$

The total amount of ordnance to be VERTREPed is found to be

$$\max(T_{31},T_{41})\left(\frac{X_{11}}{T_{11}} + K_1\right) \quad .$$

The total VERTREP time for each helicopter is

$$Z_1 = \frac{X_{31}}{T_{31}} + \sum_{j=2}^{7} \frac{X_{3j}}{T_{3j}}$$

$$Z_2 = \frac{X_{41}}{T_{41}} + \sum_{j=2}^{7} \frac{X_{4j}}{T_{4j}} \quad .$$

The total optimal VERTREP time, Z^*, is found by minimizing the maximum of Z_1 and Z_2, subject to the following constraints:

$$X_{3j} + X_{4j} = (A_j - C_j) \quad \text{for } j = 1,\cdots,7$$

$$X_{3j}\, X_{4j} = 0 \quad \text{for } j = 1,\cdots,7$$

Customer 1 must be VERTREPed by the helicopter with the larger transfer rate.

$$X_{3j}, X_{4j} \geq 0 \quad \text{for } j = 1,\cdots,7$$

The minimum Z value is found by applying the solution technique presented in Appendix A, or by investigating all the possible combinations in which the seven customers can be VERTREPed.

TRT = max(Z*,P,S) where $Z^* = \min \max (Z_1, Z_2)$

$$P = \frac{X_{11}}{T_{11}} + \sum_{j=2}^{3} \max \left(\frac{C_j}{T_{ij}}, F_j \right) + \sum_{j=1}^{3} K_j$$

$$S = \sum_{j=4}^{7} \left[\max \left(\frac{C_j}{T_{2j}}, F_j \right) + K_j \right] .$$

e. Example 4: Find the minimum total replenishment time of a seven ship task group if the CVA remains alongside the supply ship until her requirements have been fulfilled. The remaining ships break away when refueling has been completed. Two helicopters are available, but each ship is restricted to the use of one helicopter for VERTREP. The CVA is VERTREPed first by the helicopter with the larger transfer rate.

Known quantities are:

CONREP transfer rates	Same as in Example 1
VERTREP transfer rates	
Approach-rig-unrig time	
Refueling time	
Ordnance requirements	

Solution:

The CVA ordnance requirement is expressed by

$$X_{11} + 24 \left(\frac{X_{11}}{150} + 5 \right) = 500 .$$

Solving for the amount to be transferred by CONREP, X_{11} = 421 st. Therefore, the amount to be transferred by VERTREP is

$$X_{41} = 24 \left(\frac{421}{150} + 5 \right) = 79 \text{ st.}$$

The remaining equations which express ordnance requirements and CONREP upper bound values are identical to those given in Example 1.

The resulting programming problem may be stated as min Z = max (Z_1, Z_2), subject to:

$$X_{41} = 79$$

$$X_{32} + X_{41} = 30$$

$$X_{33} + X_{43} = 18$$

$$X_{34} + X_{44} = 40$$

$$X_{35} + X_{45} = 20$$

$$X_{36} + X_{46} = 8$$

$$X_{37} + X_{47} = 0$$

$$X_{3j}\, X_{4j} = 0 \quad \text{for } j = 1, \cdots, 7.$$

$$X_{3j}, X_{4j} \geq 0 \quad \text{for } j = 1, \cdots, 7.$$

Using the solution technique outlined in Appendix A,

$$\begin{array}{ll} X_{31} = 0 & X_{41} = 79 \\ X_{32} = 0 & X_{42} = 30 \\ X_{33} = 18 & X_{43} = 0 \\ X_{34} = 40 & X_{44} = 0 \\ X_{35} = 20 & X_{45} = 0 \\ X_{36} = 8 & X_{46} = 0 \\ X_{37} = 0 & X_{47} = 0 \; . \end{array}$$

Total optimal VERTREP times for helicopters 1 and 2 are

$$Z^*_1 = \frac{1}{18}(18) + \frac{1}{18}(40) + \frac{1}{18}(20) + \frac{1}{18}(8) = 4.78 \text{ hrs.}$$

$$Z^*_2 = \frac{1}{24}(79) + \frac{1}{24}(30) = 4.54 \text{ hrs.}$$

So that min Z = Z^*_1 = 4.78 hrs.

CONREP service time increases on the port side because the CVA remains alongside until her requirements have been satisfied.

$$P = \frac{421}{150} + 1.20 + 0.90 + 1.30 = 6.21 \text{ hrs.}$$

CONREP service on the starboard side does not change. S = 5.80 hrs.

TRT = max $(Z^*,P,S) = P = 6.21$ hrs.

3. Variation Three

a. Two helicopters are available. The two helicopter transfer rates are equal for each customer, i.e., $T_{3j} = T_{4j} = T_j$ for $j = 1,\cdots,7$. In addition, customer 1 is VERTREPed simultaneously by both helicopters with a combined transfer rate of T_{v1}. The remaining six customers are VERTREPed by both helicopters but not simultaneously, and therefore the individual helicopter transfer rates remain in effect.

b. In Case I, due to the fact that the helicopter transfer rates are equal and both helicopters are permitted to VERTREP the same customer, symmetry implies that both helicopters transfer equal amounts of ordnance at the optimal solution. For this same reason, Z_1 and Z_2 will have to be equal for the optimal solution. The equations which express the amount of ordnance to be VERTREPed to each customer are

$$X_{v1} = (A_1 - C_1)$$

$$X_{3j} = X_{4j} = \tfrac{1}{2}(A_j - C_j) \quad \text{for } j = 2,\cdots,7,$$

where C_j was defined in Variation One. Total optimal VERTREP time for each helicopter is given by

$$Z^* = \frac{2(A_1 - C_1)}{2T_{v1}} + \sum_{j=2}^{7} \frac{(A_j - C_j)}{2T_j} .$$

This equation contains no unknown values and so the optimal VERTREP time has been found.

The total CONREP service times are also known:

$$P = \sum_{j=1}^{3} \left[\max \left(\frac{C_j}{T_{1j}} , F_j \right) + K_j \right]$$

$$S = \sum_{j=4}^{7} \left[\max \left(\frac{C_j}{T_{2j}} , F_j \right) + K_j \right]$$

TRT = max (Z*,P,S) and the optimization problem is solved.

c. Example 5: Find the minimum total replenishment time for a seven ship task group, if all ships remain alongside the supply ship only until refueling has been completed. Two helicopters are available and the CVA is VERTREPed simultaneously by both helicopters. The remaining ships VERTREP with both helicopters but not simultaneously. Known quantities are

$\left.\begin{array}{l}\text{CONREP transfer rates}\\ \text{Approach-rig-unrig time}\\ \text{Refueling time}\\ \text{Ordnance requirements}\end{array}\right\}$ Same as in Example 1

VERTREP transfer rates:

$$T_{3j} = T_{4j} = T_j = 21 \text{ st/hr.}$$

$$T_{v1} = 30 \text{ st/hr.}$$

Solution:

The equations which express ordnance requirements and CONREP upper bound values are identical to those given in Example 1.

The helicopter transfer rates are equal and both helicopters are permitted to VERTREP the same ship; this implies that each helicopter must transfer an equal amount of ordnance to each ship. Therefore,

$X_{v1} = 138$, which implies that $X_{31} = X_{41} = 69$

$$X_{32} = X_{42} = \tfrac{1}{2}(30) = 15$$

$$X_{33} = X_{43} = \tfrac{1}{2}(18) = 9$$

$$X_{34} = X_{44} = \tfrac{1}{2}(40) = 20$$

$$X_{35} = X_{45} = \tfrac{1}{2}(20) = 10$$

$$X_{36} = X_{46} = \tfrac{1}{2}(8) = 4$$

$$X_{37} = X_{47} = 0 = 0 .$$

Total optimal VERTREP time for each helicopter is found to be

$$Z^* = \frac{1}{15}(69) + \frac{1}{21}(15) + \frac{1}{21}(9) + \frac{1}{21}(20) + \frac{1}{21}(10) + \frac{1}{21}(4) = 7.36 \text{ hrs.}$$

Total port side and starboard side CONREP service times are P = 5.81 hrs. and S = 5.80 hrs. Therefore, TRT = max $(Z^*,P,S) = Z^* = 7.36$ hrs.

d. In Case II, the amount of ordnance transferred by CONREP to customer 1 may increase because customer 1 remains alongside until all of her requirements have been satisfied. Customer 1 is VERTREPed during this time. The equation which expresses the ordnance requirement of customer 1 is

$$X_{11} + T_{v1}\left(\frac{X_{11}}{T_{11}} + K_1\right) = A_1 .$$

Solving for X_{11}, the total amount of ordnance to be CONREPed is found to be

$$X_{11} = \frac{A_1 - T_{v1} K_1}{\left(1 + \frac{T_{v1}}{T_{11}}\right)} .$$

Therefore, the total amount of ordnance VERTREPed to customer 1 is

$$X_{v1} = T_{v1} \left(\frac{X_{11}}{T_{11}} + K_1\right).$$

The amount of ordnance to be VERTREPed to the remaining customers is expressed by

$$X_{3j} = X_{4j} = \tfrac{1}{2}(A_j - C_j) \quad \text{for } j = 2,\cdots,7.$$

Total optimal VERTREP time for each helicopter is given by

$$Z^* = \frac{2}{T_{v1}} \frac{X_{v1}}{2} + \sum_{j=2}^{7} \frac{(A_j - C_j)}{2T_j} .$$

This equation contains no unknown values and so optimal VERTREP time is a known quantity. The total CONREP service times are given by

$$P = \frac{X_{11}}{T_{11}} + \sum_{j=2}^{3} \max\left(\frac{C_j}{T_{1j}}, F_j\right) + \sum_{j=1}^{3} K_j$$

$$S = \sum_{j=4}^{7} \left[\max\left(\frac{C_j}{T_{2j}}, F_j\right) + K_j\right] .$$

TRT = max(Z*,P,S) and the optimization problem is solved.

e. Example 6: Find the minimum total replenishment time of a seven ship task group, if the CVA remains alongside the supply ship until all of her requirements have been satisfied. The remaining ships break away when refueling has been completed. Two helicopters are available and the CVA is VERTREPed simultaneously by both helicopters. After the CVA has been VERTREPed, then the remaining ships VERTREP with both helicopters but not simultaneously.

Known quantities are:

$$\left.\begin{array}{l}\text{CONREP transfer rates}\\ \text{Approach-rig-unrig time}\\ \text{Refueling time}\\ \text{Ordnance requirements}\end{array}\right\}\quad\begin{array}{l}\text{Same as in}\\ \text{Example 1}\end{array}$$

VERTREP transfer rates:

$$T_{3j} = T_{4j} = T_j = 21 \text{ st/hr.}$$

$$T_{v1} = 30 \text{ st/hr.}$$

Solution:

The CVA ordnance requirement is expressed by

$$X_{11} + 30\left(\frac{X_{11}}{150} + 0.5\right) = 500.$$

Solving for the amount to be transferred by CONREP, $X_{11} = 404$ st.

Therefore, the amount to be transferred by VERTREP is

$$X_{v1} = 30\left(\frac{404}{150} + 0.5\right) = 96 \text{ st.}$$

The remaining equations which express ordnance requirements and CONREP upper bound values are identical to those given in Example 1.

The amount of ordnance to be VERTREPed to the CVA is the only change in the Z equation of Example 5. Total optimal VERTREP time for each helicopter is found to be

$$Z* = \frac{1}{15}(48) + \frac{1}{21}(15) + \frac{1}{21}(9) + \frac{1}{21}(20) + \frac{1}{21}(10) + \frac{1}{21}(4) = 5.96 \text{ hrs.}$$

CONREP service time increases on the port side because the CVA remains alongside until her requirements have been satisfied. The starboard side CONREP service time does not change ,

$$P = \frac{404}{150} + 1.20 + 0.90 + 1.30 = 6.09 \text{ hrs}$$

$S = 5.80$ hrs.

TRT = max(Z*,P,S) = P = 6.09 hrs.

4. Variation Four

a. In this situation, only one helicopter is available for VERTREP. The helicopter transfer rate is T_{3j} for $j = 1,\cdots,7$.

b. In Case I, the equation which expresses the amount of ordnance to be VERTREPed to each customer is $X_{3j} = (A_j - C_j)$ for $j = 1,\cdots,7$, where C_j was defined in Variation One. The total optimal VERTREP time is given by

$$Z = \sum_{j=1}^{7} \frac{X_{3j}}{T_{3j}} = \sum_{j=1}^{7} \frac{(A_j - C_j)}{T_{3j}} .$$

This equation contains no unknown values and so optimal VERTREP time is a known quantity. The total CONREP service times are also known.

$$P = \sum_{j=1}^{3} \left[\max\left(\frac{C_j}{T_{1j}}, F_j\right) + K_j \right]$$

$$S = \sum_{j=4} \left[\max\left(\frac{C_j}{T_{2j}}, F_j\right) + K_j \right]$$

TRT = max (Z*,P,S) and the optimization problem is solved.

c. Example 7: Find the minimum total replenishment time of a seven ship task group, if all ships remain alongside the supply ship only until refueling has been completed. One helicopter is available to VERTREP. Known quantities are

$$\left.\begin{array}{l}\text{CONREP transfer rates}\\ \text{Approach-rig-unrig time}\\ \text{Refueling time}\\ \text{Ordnance requirements}\end{array}\right\}\quad \begin{array}{l}\text{Same as in}\\ \text{Example 1}\end{array}$$

The VERTREP transfer rate is $T_{3j} = 21$ st/hr. for $j = 1,\cdots,7$.

Solution:

The equations which express ordnance requirements and CONREP upper bound values are identical to those given in Example 1. The amount of ordnance to be VERTREPed to each customer is

$$\begin{aligned} X_{31} &= 138 \\ X_{32} &= 30 \\ X_{33} &= 18 \\ X_{34} &= 40 \\ X_{35} &= 20 \\ X_{36} &= 8 \\ X_{37} &= 0 \;. \end{aligned}$$

Total optimal VERTREP time is found to be

$$\begin{aligned} Z^* &= \frac{1}{21}(138) + \frac{1}{21}(30) + \frac{1}{21}(18) + \frac{1}{21}(40) + \frac{1}{21}(20) + \frac{1}{21}(8) \\ &= 12.1 \text{ hrs.} \end{aligned}$$

Total port side and starboard side CONREP service times are

$P = 5.81$ hrs.

$S = 5.80$ hrs.

Therefore, $TRT = \max(Z^*,P,S) = Z = 12.1$ hrs.

d. In Case II, the amount of ordnance transferred by CONREP to customer 1 may increase because customer 1 remains alongside until its requirements have been satisfied. Customer 1 is VERTREPed during this period. The ordnance required by customer 1 is expressed in the following equation:

$$X_{11} + T_{31} \left(\frac{X_{11}}{T_{11}} + K_1 \right) = A_1$$

Solving for X_{11}, the total amount of ordnance to be CONREPed is found to be

$$X_{11} = \frac{A_1 - T_{31} K_1}{\left(1 + \frac{T_{31}}{T_{11}}\right)} .$$

Therefore, the total amount of ordnance to be VERTREPed is

$$X_{31} = T_{31} \left(\frac{X_{11}}{T_{11}} + K_1 \right) .$$

The amount of ordnance to be VERTREPed to the remaining customers is expressed by $X_{3j} = (A_j - C_j)$ for $j = 2,\cdots,7$. Total optimal VERTREP time is given by

$$Z^* = \frac{X_{31}}{T_{31}} + \sum_{j=2}^{7} \frac{(A_j - C_j)}{T_{31}} .$$

This equation contains no unknowns and so optimal VERTREP time is a known quantity. The total CONREP service times are

$$P = \frac{X_{11}}{T_{11}} + \sum_{j=2}^{3} \max \left(\frac{C_j}{T_{1j}} , F_j \right) + \sum_{j=1}^{3} K_j$$

$$S = \sum_{j=4}^{7} \left[\max \left(\frac{C_j}{T_{2j}} , F_j \right) + K_j \right] .$$

TRT = max (Z*,P,S) and the optimization problem is solved.

e. Example 8: Find the minimum total replenishment time of a seven ship task group, if the CVA remains alongside until all requirements have been satisfied. The remaining ships break away when refueling has been completed. One helicopter is available and it VERTREPs the CVA first. Known quantities are

$$\left.\begin{array}{l} \text{CONREP transfer rates} \\ \text{Approach-rig-unrig time} \\ \text{Refueling time} \\ \text{Ordnance requirements} \end{array}\right\} \quad \begin{array}{l} \text{Same as in} \\ \text{Example 1} \end{array}$$

The VERTREP transfer rate is $T_{3j} = 21$ st/hr.

Solution:

The CVA ordnance requirement is expressed by

$$X_{11} + 21 \left(\frac{X_{11}}{500} + .5 \right) = 500.$$

Solving for the amount to be transferred by CONREP, $X_{11} = 429$ st. Therefore, the amount to be transferred by VERTREP is

$$X_{31} = 21 \left(\frac{429}{150} + .5 \right) = 71 \text{ st}.$$

The remaining equations which express ordnance requirements and CONREP upper bound values are identical to those given in Example 1. The amount of ordnance to be VERTREPed to the CVA is the only change from the Z

equation given in Example 7. Therefore, the total optimal VERTREP time is

$$Z^* = \frac{1}{21}(71) + \frac{1}{21}(30) + \frac{1}{21}(18) + \frac{1}{21}(40) + \frac{1}{21}(20) + \frac{1}{21}(8) = 8.90 \text{ hrs.}$$

Total CONREP service time increases on the port side because the CVA remains alongside until all of her requirements have been satisfied.

$$P = \frac{429}{150} + 1.20 + 0.90 + 1.30 = 6.26 \text{ hrs.}$$

CONREP service time on the starboard side does not change.

$$S = 5.80 \text{ hrs.}$$

Therefore, TRT = max (Z*,P,S) = Z* = 8.90 hrs.

It may be remarked that if the CVA received all her ordnance requirement by CONREP, the total optimal VERTREP time would be reduced to

$$Z^* = \frac{1}{21}(30) + \frac{1}{21}(18) + \frac{1}{21}(40) + \frac{1}{21}(20) + \frac{1}{21}(8) = 5.52 \text{ hrs.}$$

CONREP service time on the port side would increase to

$$P = \frac{500}{150} + 1.20 + 0.90 + 1.30 = 6.73 \text{ hrs.}$$

The starboard side CONREP service time would not change.

$$S = 5.80 \text{ hrs.}$$

Therefore, TRT = max (Z*,P,S) = P = 6.73 hrs.

5. <u>Variation Five</u>

a. In this situation, no helicopters are available for VERTREP. All of the replenishment is conducted by CONREP, and therefore the customers must remain alongside until all of their requirements have been satisfied.

b. Case I does not apply since there are no helicopters.

c. This step does not apply since there is no Case I in this variation.

d. In Case II, the total replenishment time is found by taking the maximum of the port side and the starboard side CONREP service times. TRT = max (P,S), where

$$P = \sum_{j=1}^{3} \left[\max \left(\frac{A_j}{T_{1j}} , F_j \right) + K_j \right]$$

$$S = \sum_{j=4}^{7} \left[\max \left(\frac{A_j}{T_{2j}} , F_j \right) + K_j \right] .$$

The $\frac{A_j}{T_{1j}}$ and $\frac{A_j}{T_{2j}}$ values represent the time required for each customer to rearm via CONREP from the port side and starboard side, respectively.

e. Example 9: Find the minimum total replenishment time of a seven ship task group, if there are no helicopters available for VERTREP. All ships receive their requirements by CONREP. Known quantities are

$$\left.\begin{array}{l} \text{CONREP transfer rates} \\ \text{Approach-rig-unrig time} \\ \text{Refueling times} \\ \text{Ordnance requirements} \end{array}\right\} \quad \begin{array}{l} \text{Same as in} \\ \text{Example 1} \end{array}$$

Solution:

Total CONREP service times are given by

$$P = \frac{500}{150} + \frac{60}{25} + \frac{40}{25} + 1.30 = 8.63 \text{ hrs.}$$

$$S = \frac{70}{25} + \frac{50}{25} + \frac{30}{25} + 0.90 + 1.60 = 8.50 \text{ hrs.}$$

Therefore, TRT = max (P,S) = P = 8.63 hrs.

V. CONCLUSIONS

A. VARIATION COMPARISONS

The first variation is what the author feels is a realistic representation of a replenishment operation involving an AOE in a combat atmosphere. Since all the customer ships require fuel, they must go alongside the AOE. While alongside, these ships also receive ordnance. The AOE helicopter capability enables the customer ship to be alongside only as long as it takes to refuel. This represents the minimum amount of time each customer must spend alongside the AOE. Two helicopters VERTREP customer 1 (CVA), and one helicopter VERTREPs each of the remaining customers. This is usually done because of the need to minimize the CVA's vulnerability to attack and to enable her to continue her primary mission as soon as possible. In addition, the CVA is large enough so that the two helicopters can VERTREP simultaneously. The remaining customers are each VERTREPed by one helicopter primarily because of their low "strike down rate." Strike down rate is the rate at which material can be removed from the receiving area and placed in a storage area.

Operational data [6] which covers an 18-month period during 1968 and 1969, shows no DLG, DDG, or DD being VERTREPed by more than one helicopter when replenishing from an AOE. This same data also indicates that the use of two helicopters to simultaneously VERTREP a CVA does not double the rate at which ordnance is received. One reason for this is the CVA strike down rate. This data reflects the dependence of transfer rates upon strike down rates. Different helicopter transfer rates may occur as a result of

material deficiencies aboard the helicopter or a difference in the level of experience of each helicopter crew.

Variation Two represents another realistic situation. The principal difference from Variation One is that all customers are restricted to being VERTREPed by only one helicopter. This restriction results when the combined transfer rate for two helicopters approaches the individual helicopter transfer rate while VERTREPing the CVA. This variation also permits ordnance to reach some of the escorts by VERTREP earlier in the replenishment operation.

The solution for Variation Three may not be realistic because it is not expected that each helicopter would transfer exactly one-half of the ordnance requirement to each customer. However, this result may still be valuable because it provides a good comparison of the VERTREP time and total replenishment time with that arrived at in the first variation. The mathematics here is much simpler and the solution can be arrived at quickly.

Variations Four and Five are presented so that comparisons can be made in situations involving zero, one, and two helicopters.

The model assumes that all customers are capable of conducting CONREP and VERTREP simultaneously. It makes no distinction as to when the VERTREP takes place during the operation.

B. EXAMPLE RESULTS

The values used in the examples have been derived from operational replenishment data [6] . The same set of values is used in all the examples, so that a comparison of the results may be made. Table I lists the results of the computations.

Table I

Example Results

(Optimal Times in Minutes)

	VERTREP TIME (Z*)	CONREP TIME (MAX)	CVA ALONGSIDE TIME	CVA REPLENISH TIME	TRT
VAR I CASE I	443	349	175	285*	443
CASE II	358	365	192	192	365
VAR 2 CASE I	365	349	175	345*	365
CASE II	287	373	198	198	373
VAR 3 CASE I	442	349	175	285*	442
CASE II	358	365	192	192	365
VAR 4 CASE I	726	349	175	394*	726
CASE II	534	376	202	202	534
CASE II**	331	404	230	230	404
VAR 5 CASE II	-	518	230	230	518

*This value is meaningful only when CONREP and VERTREP start at the same time.

**CVA receives all ordnance by CONREP.

In Variation One while the Case II situation requires the CVA to be alongside 17 minutes more than in Case I, it reduces the VERTREP time by 95 minutes, and total replenishment time by 78 minutes.

There was an unexpected occurrence in Variation Two, where the Case II total replenishment time was larger than Case I total replenishment time. This occurred primarily because the CONREP time and the total VERTREP time in Case I were nearly the same (349 minutes and 365 minutes, respectively). The additional CONREP time increased the total replenishment time, so that it was greater than the Case I situation.

The results of Variation Three for this particular set of data are nearly identical to those found in Variation One. The VERTREP transfer rate used in these examples was found by taking the average of the two helicopter transfer rates given in Variation One examples.

Comparing the zero, one, and two helicopter situations for Case I shows that the use of either one or two helicopters reduces total CONREP time by 169 minutes and CVA alongside time by 55 minutes. The use of two helicopters reduces total replenishment time by 75 minutes and 153 minutes respectively for Variation One and Variation Two. However, in going from zero to one helicopter, total replenishment time actually increases by 208 minutes. This is a result of all ships breaking away when refueling is completed and then receiving the remaining ordnance by VERTREP from a single helicopter.

In Case II, the comparison of zero, one, and two helicopter situations shows that the introduction of one helicopter reduces CONREP time by 142 minutes. The introduction of two helicopters reduces the CONREP time by 145 minutes in Variation One and by 153 minutes in Variation Two. Total replenishment time increases by 116 minutes with the introduction of one helicopter; it reduces by 153 minutes in Variation One and 145 minutes in Variation Two with the introduction of two helicopters. If the CVA receives all her ordnance by CONREP, Variation Four CONREP time increases by 28

minutes; however, total replenishment time decreases by 130 minutes over the regular Case II situation.

The example results for this particular set of data indicate the effect of the optimal employment of helicopters in reducing total CONREP time, CVA alongside time, and total replenishment time. The examples also illustrate that even an optimal employment of helicopters may lead to an undesirable situation. For example, in Variation Four the Case I situation shows the VERTREP time to be 726 minutes, which is considerably larger than the CONREP time. Further, it is longer than a single helicopter could be expected to operate. By going to a Case II situation and having the CVA completely rearm by CONREP, total replenishment time decreases by 322 minutes while total CONREP time increases by only 55 minutes. VERTREP time reduces by 395 minutes.

Under certain circumstances reducing total replenishment time may be the primary concern, while under other circumstances it may be more important to reduce the CVA alongside time even if this causes an increase in total replenishment time. Comparing the results of different cases shows which case is best suited to a particular set of circumstances.

C. EXTENSIONS

This paper represents a first attempt at analytically exploring the replenishment operation where both CONREP and VERTREP are involved. One extension in this area might involve expanding to more supply ships. For example, one could consider a replenishment operation involving two supply ships, an AO and an AE, or an AO and an AOE. In both of these cases, if the ordnance requirements could be satisfied by VERTREP, then the customer alongside the AO would not have to go alongside the AE or the AOE.

Another extension may involve an AFS (stores and provisions ship) in which case customers would not have to go alongside at all, providing they could satisfy their requirements by VERTREP.

APPENDIX A

A Modified Linear Programming Solution Technique for Solving Problems of the Form $Z = \min \max (Z_1, Z_2)$

Let Z_1 and Z_2 be two linear functions of the form:

$$Z_1 = C_1X_1 + C_2X_2 + C_3X_3 + \cdot \cdot \cdot + C_mX_m$$

$$Z_2 = D_1Y_1 + D_2Y_2 + D_3Y_3 + \cdot \cdot \cdot + D_mY_m \, .$$

The problem is to find the minimum of the maximum of Z_1 and Z_2, subject to the following constraints:

$$X_1 + Y_1 = b_1$$

$$X_2 + Y_2 = b_2$$

$$\vdots$$

$$X_m + Y_m = b_m$$

$$X_1 \; Y_1 = 0$$

$$\vdots$$

$$X_m \; Y_m = 0$$

$$X_j, \; Y_j \geq 0 \quad \text{for } j = 1, \cdots, m \, .$$

The solution is developed below, using a modified linear programming technique.

Let R_1 be the region where $Z_1(X) \leq Z_2(Y)$ and R_2 be the region where $Z_1(X) > Z_2(Y)$. Then $R_1 \cap R_2 = \emptyset$ and $R_1 \cup R_2$ equals the whole space. The method of solution of the original problem consists of finding the optimal

solution in R_1 as well as in R_2, and then selecting that solution of the two for which Z is smaller; therefore, the following two problems:

1. $\min Z_2 = D_1Y_1 + D_2Y_2 + \cdots + D_mY_m$

subject to

(A1) $X_1 + Y_1 = b_1$

(A2) $X_2 + Y_2 = b_2$

$\vdots$

(Am) $X_m + Y_m = b_m$

(A(m+1)) $X_1\ Y_1 = 0$

$\vdots$

(A2m) $X_m\ Y_m = 0$

(A(2m+1)) $Z_1(X) - Z_2(Y) \leq 0$

$X_j, Y_j \geq 0$ for $j = 1, \cdots, m$.

2. $\min Z_1 = C_1X_1 + C_2X_2 + \cdots + C_mX_m$

subject to

(A(2m+2)) $X_1 + Y_1 = b_1$

(A(2m+3)) $X_2 + Y_2 = b_2$

$\vdots$

(A(3m+1)) $X_m + Y_m = b_2$

(A(3m+2)) $X_1\ Y_1 = 0$

$\vdots$

(A(4m+1)) $X_m\ Y_m = 0$

(A(4m+2)) $Z_1(X) - Z_2(Y) > 0$

$X_j, Y_j \geq 0$ for $j = 1, \cdots, m$.

STEP 1: Write equation (A(2m+1)) in terms of X_1 through X_m by substitution of equations (A1) through (Am) into (A(2m+1)).

$$(A(2m+1)') \qquad (C_1 + D_1) X_1 + (C_2 + D_2) X_2 + \cdots + (C_m + D_m) X_m \leq \sum_{j=1}^{m} b_j .$$

STEP 2: Construct the initial simplex tableau using equations (A1) through (Am) and (A(2m+1)'). The initial basis variables are $Y_1, \cdots, Y_m$, Y_{m+1}, where Y_{m+1} is the slack variable for equation (A(2m+1)'). The slack variable must always be a member of the basis in view of the non-linear constraints, equations (A(m+1)) through (A2m); otherwise, one of the $X_j Y_j = 0$ conditions may be violated. This condition causes the use of restricted basis entry.

STEP 3: Perform iterations to minimize Z_2, keeping the slack variable non-negative. This guarantees that the problem remains within the assigned region.

STEP 4: A solution to the problem has been achieved if there is still a $Z_j - C_j > 0$, and the only variable which may enter the basis would violate the non-linear constraints. This solution is $Z_2(Y_B) = \min Z_2(Y)$ and $Z_2(Y_B) \geq Z_1(X_B)$ where X_B and Y_B are the X and Y variables in the final basis.

There may be a variety of choices of variables which can be introduced into the basis to reach this position. Special care must be taken when choosing the variable to be used. Blindly following the above steps may lead to a non-optimal solution. The criterion used is to make the Z value as small as possible while maintaining the slack variable at a non-negative level.

STEP 5: Perform the same steps 1 through 4 on Problem 2, changing the notation appropriately. In obtaining a solution, attempt to reach a minimum which is less than or equal to the solution found in Problem 1, $(Z_2(Y_B))$. The solution in Problem 2 is $Z_1(X_B') = \min Z_1(X)$ and $Z_1(X_B') \geq Z_2(Y_B')$ where X_B' and Y_B' are the X and Y variables in the final basis.

STEP 6: Compare $Z_1(X_B')$ and $Z_2(Y_B)$. If $Z_1(X_B') \leq Z_2(Y_B)$ then $Z_1(X_B') = \min \max (Z_1, Z_2)$ and if $Z_1(X_B') > Z_2(Y_B)$ then $Z_2(Y_B) = \min \max (Z_1, Z_2)$.

EXAMPLE:

$$\min Z = \max (Z_1, Z_2)$$

$$Z_1 = 4X_1 + 3X_2 + 3X_3 + 2X_4 + 3X_5 + 3X_6 + 3X_7$$

$$Z_2 = 3Y_1 + 2Y_2 + 2Y_3 + 2Y_4 + 2Y_5 + 2Y_6 + 2Y_7$$

subject to

$$X_1 + Y_1 = 40$$

$$X_2 + Y_2 = 35$$

$$X_3 + Y_3 = 30$$

$$X_4 + Y_4 = 20$$

$$X_5 + Y_5 = 25$$

$$X_6 + Y_6 = 10$$

$$X_7 + Y_7 = 15$$

$$X_1 Y_1 = 0$$

$$X_2 Y_2 = 0$$

$$\vdots$$

$X_7 \; Y_7 = 0$

$X_j \, , Y_j \geq 0$ for $j = 1, \cdots, 7$.

Separate the original problem into two regions to obtain two problems.

Problem 1: $Z_1(X) \leq Z_2(Y)$

$$\min Z_2 = 3Y_1 + 2Y_2 + 2Y_3 + 2Y_4 + 2Y_5 + 2Y_6 + 2Y_7$$

subject to

(1) $X_1 + Y_1 = 40$

(2) $X_2 + Y_2 = 35$

(3) $X_3 + Y_3 = 30$

(4) $X_4 + Y_4 = 20$

(5) $X_5 + Y_5 = 25$

(6) $X_6 + Y_6 = 10$

(7) $X_7 + Y_7 = 15$

(8) $X_1 \; Y_1 = 0$

.
.
.

(14) $X_7 \; Y_7 = 0$

(15) $Z_1(X) - Z_2(Y) \leq 0$

$X_j \, , Y_j \geq 0$ for $j = 1, \cdots, 7$.

Problem 2: $Z_1(X) > Z_2(Y)$

$$\min Z = 4X_1 + 3X_2 + 3X_3 + 2X_4 + 3X_5 + 3X_6 + 3X_7$$

subject to

(16) $X_1 + Y_1 = 40$

(17) $X_2 + Y_2 = 35$

(18) $X_3 + Y_3 = 30$

(19) $X_4 + Y_4 = 20$

(20) $X_5 + Y_5 = 25$

(21) $X_6 + Y_6 = 10$

(22) $X_7 + Y_7 = 15$

(23) $X_1 \; Y_1 = 0$

$\vdots$

(29) $X_7 \; Y_7 = 0$

(30) $Z_1(X) - Z_2(Y) > 0$

$X_j \, , Y_j \geq 0$ for $j = 1, \ldots, 7$.

STEP 1: In Problem 1, writing equation (15) in terms of the X's,

(15') $7X_1 + 5X_2 + 5X_3 + 4X_4 + 5X_5 + 5X_6 + 5X_7 \leq 390$.

STEP 2: The initial simplex tableau with Y_1 through Y_8 comprising the initial basis where Y_8 is the slack variable for equation (15').

	0	0	0	0	0	0	0	0	3	2	2	2	2	2	2	0
	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_8	40	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
a_9	35	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0
a_{10}	30	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0
a_{11}	20	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0
a_{12}	25	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
a_{13}	10	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0
a_{14}	15	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0
a_{15}	390	7	5	5	4	5	5	5	0	0	0	0	0	0	0	1
$Z_j - D_j$	390	3	2	2	2	2	2	2	0	0	0	0	0	0	0	0

STEP 3: Starting arbitrarily with the largest $Z_j - D_j$ value, introduce a_1 into the basis. Compare $\frac{390}{7}$ and $\frac{40}{1}$. Min $\theta = \frac{40}{1}$. Therefore, remove a_8 from the basis.

	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_1	40	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
a_9	35	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0
a_{10}	30	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0
a_{11}	20	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0
a_{12}	25	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
a_{13}	10	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0
a_{14}	15	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0
a_{15}	110	0	5	5	4	5	5	5	-7	0	0	0	0	0	0	1
$Z_j - D_j$	270	0	2	2	2	2	2	2	-3	0	0	0	0	0	0	0

If the min θ criteria and the restriction that a_{15} must remain in the basis is satisfied, then a_4, a_6, a_7 are the only choices available. Choose a_4 since it will reduce Z by the greatest amount and will remove a_{11} from the basis.

	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_1	40	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
a_9	35	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0
a_{10}	30	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0
a_4	20	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0
a_{12}	25	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
a_{13}	10	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0
a_{14}	15	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0
a_{15}	30	0	5	5	0	5	5	5	-7	0	0	-4	0	0	0	1
$Z_j - D_j$	230	0	2	2	0	2	2	2	-3	0	0	-2	0	0	0	0

The only way to change the basis at this point would be for a_{15} to be removed. Therefore, a solution has been reached.

$Z_2 = 230 \quad Y_B = (Y_2, Y_3, Y_5, Y_6, Y_7)$

This solution could have been reached with $Y_B = (Y_1, Y_3, Y_5)$ or $Y_B = (Y_1, Y_3, Y_6, Y_7)$.

STEP 4: Min $Z_2(Y) = Z_2(Y_B) = 230$ and $Z_2(Y_B) \geq Z_1(X)$. $230 > 200$.

STEP 5: Repeat steps 1 through 4. In Problem 2, writing equation (30) in terms of the Y's and multiplying this equation by -1 yields

(30') $7Y_1 + 5Y_2 + 5Y_3 + 4Y_4 + 5Y_5 + 5Y_6 + 5Y_7 < 545$.

The initial simplex tableau with X_1 through X_8 comprising the initial basis where X_8 is the slack variable for equation (30').

		4	3	3	2	3	3	3	0	0	0	0	0	0	0	0
	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_1	40	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
a_2	35	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0
a_3	30	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0
a_4	20	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0
a_5	25	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
a_6	10	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0
a_7	15	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0
a_{15}	545	0	0	0	0	0	0	0	7	5	5	4	5	5	5	1
$Z_j - C_j$	545	0	0	0	0	0	0	0	4	3	3	2	3	3	3	0

Starting again with the largest $Z_j - C_j$ value, introduce a_8 into the basis. Compare $\frac{545}{7}$ and $\frac{40}{1}$. Min $\theta = \frac{40}{1}$. Therefore, remove a_1 from the basis.

	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_8	40	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
a_2	35	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0
a_3	30	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0
a_4	20	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0
a_5	25	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
a_6	10	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0
a_7	15	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
a_{15}	265	-7	0	0	0	0	0	0	0	5	5	4	5	5	5	1
$Z_j - C_j$	385	-4	0	0	0	0	0	0	0	3	3	2	3	3	3	0

a_9 through a_{14} may enter the basis without violating the given restrictions. Therefore, again choosing from the largest $Z_j - C_j$, introduce a_9 and remove a_2 from the basis.

	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_8	40	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
a_9	35	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0
a_3	30	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0
a_4	20	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0
a_5	25	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
a_6	10	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0
a_7	15	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0
a_{15}	90	-7	-5	0	0	0	0	0	0	0	5	4	5	5	5	1
$Z_j - C_j$	280	-4	-3	0	0	0	0	0	0	0	3	2	3	3	3	0

Now only a_{11}, a_{13}, a_{14} may enter the basis. Choosing the one which reduces Z by the greatest amount, introduce a_{14} and remove a_7 from the basis.

	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_8	40	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
a_9	35	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0
a_3	30	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0
a_4	20	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0
a_5	25	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
a_6	10	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0
a_{14}	15	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0
a_{15}	15	-7	-3	0	0	0	0	-5	0	0	5	4	5	5	0	1
$Z_j - C_j$	235	-4	-3	0	0	0	0	-3	0	0	3	2	3	3	0	0

The only way to change the basis at this point would be for a_{15} to be removed. Therefore, a solution has been reached.

$$Z_1(X_B') = 235 \quad X_B' = (X_3, X_4, X_5, X_6)$$

$$\min Z_1(X) = Z_1(X_B') = 235 \text{ and}$$

$$Z_1(X_B') \geq Z_2(Y') \qquad 235 > 220.$$

STEP 6: Compare $Z_1(X_B')$ and $Z_2(Y_B)$

$$Z_1(X_B') = 235 \text{ and } Z_2(Y_B) = 230$$

$$Z_2(Y_B) < Z_1(X_B') \text{ implies that } Z_2(Y_B) = \min\max(Z_1, Z_2).$$

Therefore, $Z_1 = 4X_1 + 2X_4 = 200$ and

$$Z_2 = 2Y_2 + 2Y_3 + 2Y_5 + 2Y_6 + 2Y_7 = 230.$$

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13. ABSTRACT

This is a model of the underway replenishment of a task group by a single supply ship which is capable of transferring logistic items by helicopter as well as by the connected method. The model considers two cases where replenishment time is minimized. In one case all ships break away from the supply ship when refueling is complete. In the other case, the CVA remains alongside until all her requirements have been satisfied while the remaining ships break away when refueling is complete.

The replenishment operation discussed deals specifically with a task group composed of one CVA, three DLG's and three DD's being rearmed and refueled by a single AOE. The specific portions of ordnance received via connected replenishment and vertical replenishment for each ship are the unknown quantities to be determined, while the transfer rates, refueling times, and total ordnance requirement are assumed to be known. A modified linear programming technique is used to determine an optimal employment of helicopters so that vertical replenishment time, and so the total replenishment time, is minimized. Operational data is used to establish the transfer rates and the individual ship requirements.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Vertical Replenishment						
Underway Replenishment						
Replenishment at Sea						

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